Lecture Notes for Chapter 8

International Financial Markets and Institutions

Chapter 8

Portfolio theories of exchange rate behaviour III

Harjoat S. Bhamra

Road Map

12 Summary of international finance

13 Revision classes

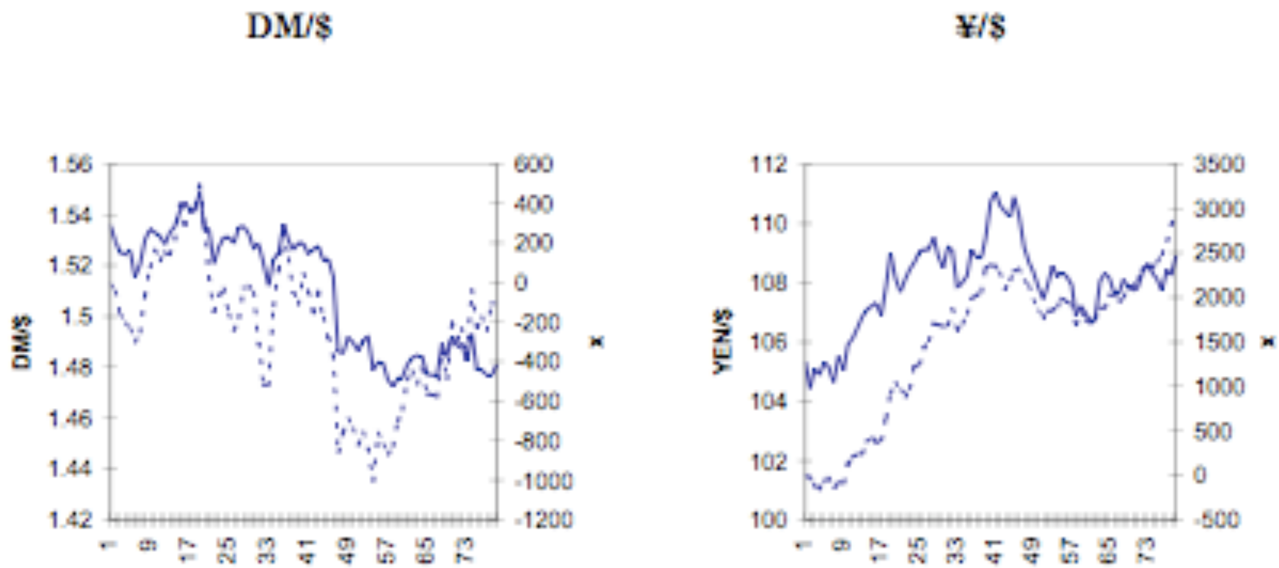
8.6 Mean-variance portfolio choice theory of the exchange rate

8.6.1 Motivation

Figure 1

Four Months of Exchange Rates (solid) and Order Flow (dashed)

May 1-August 31, 1996



8.6.2 Aim and appoach

- We want to determine the USD/GBP exchange rate, S.

- Use standard mean-variance portfolio theory to determine the demand for UK riskfree bonds.

- If we know the GBP value of the UK riskfree bonds, then know their supply.

- The USD/GBP spot rate adjusts so demand equals supply

8.6.3 Assumptions and basic model

- Assume all investors in the world want to hold the same portfolio and that they have the same risk aversion, θ.
- Take USD are our reference currency.
- Investors choose a portfolio, which is a mixture of riskfree US bonds, riskfree UK bonds and an equity portfolio.

- Riskfree US bonds earn the US riskfree rate r_0.
- Riskfree UK bonds are risky when measured in USD terms—their return is given by $\tilde{r}_1$.
- The equity portfolio has return in USD terms of $\tilde{r}_2$.
- If net world wealth is W, it will be split between the above three assets in the following proportions:

  w_0, w_1, and w_2, where

$$w_0 + w_1 + w_2 = 1.$$

- Once we have picked w_1 and w_2, we know w_0.
- Choice of w_1 and w_2 defines a specific portfolio.

- The return on that portfolio is given by

$$\tilde{r}_p = w_0 r_0 + w_1 \tilde{r}_1 + w_2 \tilde{r}_2$$

- Want to choose w_1 and w_2 optimally.

8.6.4 Finding the optimal demand

- Each asset contributes a *benefit* to an investor: its own expected return to the portfolio's expected return, and a *cost*: its own portfolio covariance risk to the total variance risk of the portfolio.

- From microeconomics we know that the investor's optimum portfolio is achieved at the point where the **ratio of the marginal benefit from the expected return to the to marginal cost from the covariance is equal across all assets**.

- The marginal benefit of investing more in the riskfree UK bond is its expected return relative to the US riskfree bond, i.e. $\mathrm{E}[\tilde{r}_1 - r_0]$.

- The marginal cost of investing more in the riskfree UK bond is that asset's contribution to the variance of the portfolio, i.e. $\mathrm{cov}(\tilde{r}_1, \tilde{r}_p)$.

- If the marginal benefit-marginal cost ratio is equal across assets for the optimal portfolio, then

$$\frac{\mathrm{E}(\tilde{r}_1 - r_0)}{\mathrm{cov}(\tilde{r}_1, \tilde{r}_p)} = \frac{\mathrm{E}(\tilde{r}_2 - r_0)}{\mathrm{cov}(\tilde{r}_2, \tilde{r}_p)} = \theta$$

- It turns out that the marginal benefit-marginal cost ratio is equal to risk aversion.

  Rerranging the above equation:

$$\begin{aligned} \mathrm{E}(\tilde{r}_1 - r_0) &= \theta \mathrm{cov}(\tilde{r}_1, \tilde{r}_p) \\ \mathrm{E}(\tilde{r}_2 - r_0) &= \theta \mathrm{cov}(\tilde{r}_2, \tilde{r}_p) \end{aligned}$$

- In the above equations, we can express $\mathrm{cov}(\tilde{r}_1, \tilde{r}_p)$ and $\mathrm{cov}(\tilde{r}_2, \tilde{r}_p)$ explicitly in terms of the portfolio weights w_1 and w_2.

$$\begin{aligned}
\mathrm{E}(\tilde{r}_1 - r_0) &= \theta[w_1 \mathrm{var}(\tilde{r}_1) + w_2 \mathrm{cov}(\tilde{r}_1, \tilde{r}_2)] \\
\mathrm{E}(\tilde{r}_2 - r_0) &= \theta[w_2 \mathrm{var}(\tilde{r}_2) + w_1 \mathrm{cov}(\tilde{r}_1, \tilde{r}_2)]
\end{aligned}$$

- Solving these two equations, we get the solution for the optimal portfolio weights, w_1 and w_2.

$$\begin{aligned}
w_1 &= \frac{(\mathrm{E}[\tilde{r}_1] - r_0)\mathrm{var}[\tilde{r}_2] - (E[\tilde{r}_2] - r_0)\mathrm{cov}(\tilde{r}_1, \tilde{r}_2)}{\theta[\mathrm{var}(\tilde{r}_1)\mathrm{var}(\tilde{r}_2) - (\mathrm{cov}(\tilde{r}_1, \tilde{r}_2))^2]} \\
w_2 &= \frac{(\mathrm{E}[\tilde{r}_2] - r_0)\mathrm{var}[\tilde{r}_1] - (E[\tilde{r}_1] - r_0)\mathrm{cov}(\tilde{r}_1, \tilde{r}_2)}{\theta[\mathrm{var}(\tilde{r}_1)\mathrm{var}(\tilde{r}_2) - (\mathrm{cov}(\tilde{r}_1, \tilde{r}_2))^2]}
\end{aligned}$$

8.6.5 Equilibrium in the world capital market

- The equilibrium price of each asset is the one that makes the aggregate demand for the asset equal to its total supply.

- Remember total wealth worldwide is W.

- Then, the amount invested in US bonds is $(w_0 \times W)$, in UK bonds is $(w_1 \times W)$, and in the equities portfolio is $(w_2 \times W)$.

- Suppose that net value in GBP of the UK bonds is B^*. Then, the equilibrium spot rate is the one which makes the demand for this bond equal to its supply: $w_1 \times W = S \times B^*$, which implies that

$$S = (w_1 \times W)/B^*.$$

Example 8.1 (Optimal portfolio weights)

Suppose that we are given the following information about the one-period asset returns: the excess return on the UK riskfree bond is 0.150 and on the risky asset is 0.045 p.a. The variance of returns on the UK riskfree bond is 0.04 and that of the equities portfolio is 0.01, while the covariance between these two assets is 0.01. The degree of risk aversion is 5.

– To find the optimal portfolio with a risk aversion θ equal to 5, we plug the information that we have into the two equations:

$$
\begin{aligned}
0.150 &= 5[(0.04)w_1 + 0.01w_2] \\
0.045 &= 5[(0.01)w_2 + 0.01w_1]
\end{aligned}
$$

– Solving the above equations for w_1 and w_2, we get that $w_1 = 0.70$, $w_2 = 0.20$, and the investment in the riskfree asset is $1 - w_1 - w_2 = 1 - 0.70 - 0.20 = 0.10$.

– Thus, the composition of the portfolio that contains only risky assets (using the USD as our reference currency) can be obtained by computing the relative weights of the UK bond and the equities portfolio:

  - ⋆ the weight of the UK bond in this portfolio is $0.7/(0.7{+}0.2) = 0.78$, and
  - ⋆ the weight of the equities portfolio is $0.2/(0.7{+}0.2) = 0.22$.

Example 8.2 (Equilibrium)

- All investors are identical
- The total portfolio wealth across all investors, W, is USD 100 billion.
- The supply of the US riskless asset is 2 billion units
- The supply of equities is 1 billion shares
- The number of UK unit bonds outstanding is 4 billion.
- The net value of UK bonds outstanding is GBP 40 B.
- Also, assume that the optimal portfolio weights are those computed in the previous example.

– In the previous example, the portfolio weight for UK bonds is 70%. If net wealth is USD 100 B, then
the demand for UK bonds is $0.70 \times \text{USD } 100\text{B} = \text{USD } 7 \text{ B}$.

– Similarly,
the demand for US bonds is $0.10 \times \text{USD } 100\text{B} = \text{USD } 10 \text{ B}$, and
the demand for equities is 20% of USD 100 B = USD 20 B.

– The amount of wealth invested in the domestic bond (asset 0) is $0.10 \times \text{USD } 100\text{B} = \text{USD } 10 \text{ B}$, so the price of each unit of the domestic riskfree asset must be $\text{USD } 10 \text{ B}/2 \text{ B} = \text{USD } 5$.

– Similarly, the amount invested in the UK bond is $0.70 \times \text{USD } 100\text{B} = \text{USD } 70 \text{ B}$

- If the net value of UK bonds outstanding is $\text{GBP } 40 \text{ B}$, then the the exchange rate is $\text{USD } 70 \text{ B}/\text{GBP } 40 \text{ B} = \text{USD}/\text{GBP } 1.75$.

- Finally, the amount invested in equities is $0.20 \times \text{USD } 100\text{B} = \text{USD } 20 \text{ B}$; thus, the price of one share is $\text{USD } 20 \text{ B}/1 \text{ B} = \text{USD } 20$.